

# OIST Workshop

## Geometry and Topology of 3-manifolds workshop

May 25 - May 28, 2018  
OIST Seaside House

Organizers:

Joel Hass, University of California, Davis, USA  
Jessica Purcell, Monash University, Melbourne, Australia  
Anastasiia Tsvietkova, OIST, Japan/ Rutgers University, Newark, USA

Local Co-Organizers (OIST)

Tirasan Khandhawit, Dale Koenig, Nicholas Owad, Robert Tang

This workshop is supported by OIST

## Schedule

### Thursday, May 24

There will be a special public lecture by Joel Hass on OIST campus followed by tea. The location will be at the room B250, Level B, Central Building.

16:00–17:00 Public lecture by **Joel Hass (UC Davis)**

*Comparing shapes of genus zero*

17:00–17:30 Tea time (Central Building, floor B, room B253)

### Friday, May 25

10:00–10:50 **Oliver Dasbach (Louisiana State University)**

*The Colored Jones Polynomial,  $q$ -series identities, and the hyperbolic volume*

11:10–12:00 **David Gabai (Princeton University)**

*Small cusped hyperbolic 3-manifolds*

12:00–13:20 Lunch

13:20–14:10 **Eiko Kin (Osaka University)**

*A construction of pseudo-Anosov braids with small normalized entropies*

14:30–15:20 **Hyam Rubinstein (University of Melbourne)**

*Counting incompressible surfaces in 3-manifolds*

15:40–16:30 **Tali Pinsky (Technion - Israel Institute of Technology)**

*An upper bound for volumes of geodesics*

16:45–17:00 **Robert Baughman (Executive Vice President, OIST)**

17:00–Dinner

## Saturday, May 26

9:30–10:20 **Abigail Thompson (UC Davis)**

*Trisections and link surgeries*

10:40–11:30 **Chuichiro Hayashi (Japan Women's University)**

*2-spheres in Morse positions with respect to the open-book decomposition of the 3-sphere*

11:30–12:15 lunch

12:15–17:30 excursion (or free time)

17:30– Dinner

## Sunday, May 27

10:00–10:50 **Sadayoshi Kojima (Tokyo Institute of Technology)**

*Quasi-Isometry of Invariants*

11:10–12:00 **Makoto Sakuma (Hiroshima University)**

*Kleinian groups generated by two parabolic transformations*

12:00–13:20 Lunch

13:20–14:10 **Stephan Tillmann (University of Sydney)**

*Structure of minimal triangulations*

14:30–15:20 **Benjamin Burton (University of Queensland)**

*From parameterised to non-parameterised algorithms in knot theory*

15:40–16:30 **Marc Culler (University of Illinois, Chicago)**

*Computation of Character Varieties*

17:00–Banquet

## Monday, May 28

9:00–9:50 **Autumn Kent (University of Wisconsin)**

*Skinning maps along thick rays*

10:10–11:00 **Ana Lecuona (l'Universite d'Aix-Marseille)**

*The slope of a link: computation and applications.*

11:20–12:10 **Josh Howie (Monash University)**

*Geometry of alternating links on surfaces*

12:10–13:10 Lunch

13:10–14:00 **Yoav Moriah (Technion - Israel Institute of Technology)**

*Diagram uniqueness for highly twisted Knots*

17:00– Dinner

# Abstract

## **Benjamin Burton (University of Queensland)**

*From parameterised to non-parameterised algorithms in knot theory*

We describe some recent developments in treewidth-based algorithms for knots, which exploit the structure of the underlying 4-valent planar graph. In particular, we show how these led to the first general sub-exponential-time algorithm for the HOMFLY-PT polynomial, and we describe some recent progress on parameterised algorithms for unknot recognition.

## **Marc Culler (University of Illinois, Chicago)**

*Computation of Character Varieties*

The talk will describe a collection of computational tools for studying  $SL(2, \mathbb{C})$ ,  $SL(2, \mathbb{R})$  and  $SU(2)$  character varieties of general knot manifolds. We will describe these tools, and discuss how the computations work and what information they can provide about a 3-manifold.

## **Oliver Dasbach (Louisiana State University)**

*The Colored Jones Polynomial,  $q$ -series identities, and the hyperbolic volume*

*We discuss  $q$ -series that are associated to the colored Jones polynomial of alternating links, and identifies among those. In particular, we compare the behavior of the  $q$ -series under certain generalized knot sums to the behavior of the hyperbolic volumes of those knots.*

## **David Gabai (Princeton University)**

*Small cusped hyperbolic 3-manifolds.*

We show that all 1-cusped hyperbolic 3-manifolds with a maximal cusp of volume at most 2.62 is obtained by filling one of a small number of explicit two- and three-cusped hyperbolic 3-manifolds. Joint work with Robert Haraway, Robert Meyerhoff, Nathaniel Thurston and Andrew Yarmola.

## **Chuichiro Hayashi (Japan Women's University)**

*2-spheres in Morse positions with respect to the open-book decomposition of the 3-sphere*

Let  $H$  be the simplest open-book decomposition of the 3-sphere  $S^3$ . A 2-sphere  $S$  in  $S^3$  is naturally decomposed into quadrilaterals if it is in a Morse position with respect to  $H$  and has no circle as a component of intersection with any page of  $H$ . We give a necessary and sufficient condition for a quadrilateral decomposition of a 2-sphere to be naturally derived from a Morse position as above. Moreover, for an arbitrary natural number  $n$ , we give a concrete example of 2-sphere in a Morse position as above which has a vertex of valency  $n$ . (joint work with MISAKI KATOU, MIKI NESORI, HIKARI TAKANO)

**Joel Hass (UC Davis)***Comparing shapes of genus zero*

Almost everything we encounter in our 3-dimensional world is a surface - the outside of a solid object. Moreover there is an explosive increase in the availability of digitized representations of surfaces in 3D. Comparing the shapes of surfaces is, not surprisingly, a fundamental problem in both theoretical and applied mathematics. Facial recognition, drug design and much of radiology, for example, are concerned with comparing and aligning surfaces. Deep mathematical results are now being used to study objects such as bones, brain cortices, proteins and biomolecules by studying their surface geometry. This talk will discuss recent joint work with Patrice Koehl that introduces a new way to align and compare surfaces, and how well it performs relative to other methods and to human experts.

**Josh Howie (Monash University)***Geometry of alternating links on surfaces*

We study links in 3-manifolds which have alternating diagrams onto orientable surfaces of positive genus. When the diagram is sufficiently complicated, we are able to obtain topological and geometrical information about the link exterior. In particular, we can tell if the link is hyperbolic and obtain bounds on volume, know whether the checkerboard surfaces are essential or quasi-fuchsian, and rule out exceptional Dehn fillings. Joint work with Jessica Purcell.

**Autumn Kent (University of Wisconsin)***Skinning maps along thick rays*

I'll discuss work in progress with K. Bromberg and Y. Minsky. We show that the diameter of the skinning map of an acylindrical 3-manifold along a thick ray in the Teichmueller space is bounded by constants depending only on the injectivity radius and genus of the boundary.

**Eiko Kin (Osaka University)***A construction of pseudo-Anosov braids with small normalized entropies*

Pseudo-Anosov mapping classes define numerical invariants, the entropy and the normalized entropy. The latter invariant is suited for the context of hyperbolic 3-manifolds. A braid with  $n$  strands can be identified with a mapping class on the  $n$ -punctured disk, and pseudo-Anosov braids and their normalized entropies make sense. We consider pseudo-Anosov braids whose braid permutations have fixed points, and consider the mapping tori of those braids. The magic 3-manifold, the exterior of the 3-chain link is the simplest example in such class of hyperbolic 3-manifolds. Given a pseudo-Anosov braid  $b$  whose permutation has a fixed point, we provide a construction of many sequences of pseudo-Anosov braids with small normalized entropies whose mapping tori are homeomorphic to the mapping torus of the initial braid  $b$ . This is joint work with Susumu Hirose.

**Sadayoshi Kojima (Tokyo Institute of Technology)***Quasi-Isometry of Invariants*

We introduce the notion of quasi-isometry for invariants of some objects such as 3-manifolds with values in metric spaces. This notion is stronger than comparability for real valued invariants. Then we would like to discuss some examples and variants.

**Ana Lecuona (l'Universite d'Aix-Marseille)**

*The slope of a link: computation and applications.*

In this talk we will introduce an invariant, the slope, for a colored link in a homology sphere together with a suitable multiplicative character defined on the link group. The slope takes values in the complex numbers union infinity and it is real for finite order characters. It is a multivariable generalization of Kojima-Yamasaki eta-function and can be expressed as a quotient of Conway polynomials. We will focus on how to compute this invariant via C-complexes and we will give several applications. This is a work in progress with Alex Degtyarev and Vincent Florens.

**Yoav Moriah (Technion - Israel Institute of Technology)**

*Diagram uniqueness for highly twisted Knots*

*Every knot has a plat projection, obtained by closing up a braid with bridges. The plat projection is determined by the number of strands and the number of rows of twist regions in the braid, and an integer number of crossings in each twist region. We show that under certain restrictions, namely, that each of their twist regions contains at least three crossings, and their length is sufficiently long with respect to their width, a plat satisfying these conditions is unique. Thus these plats classify the respective knots.*

**Tali Pinsky (Technion - Israel Institute of Technology)**

*An upper bound for volumes of geodesics*

*Consider a closed geodesic  $\gamma$  on a hyperbolic surface  $S$ , embedded in the unit tangent bundle of  $S$ . If  $\gamma$  is filling its complement is a hyperbolic three manifold, and thus has a well defined volume. I will discuss how to use Ghys' template for the geodesic flow on the modular surface to obtain an upper bound for this volume in terms of the length of  $\gamma$ . This is joint work with Maxime Bergeron and Lior Silberman.*

**Hyam Rubinstein (University of Melbourne)**

*Counting incompressible surfaces in 3-manifolds*

This is joint work with Nathan Dunfield, Stavros Garafoulidis and Craig Hodgson and is an outgrowth of our big project on understanding the 3d index.

We give an algorithm to describe all normal representatives of a given incompressible surface in a 3-manifold with a 1-efficient triangulation. We focus on the case of ideal triangulations and indicate the modifications required for the closed case.

The algorithm has two parts. In the first a graph is constructed where the vertices are the normal representatives of the isotopy class of a given incompressible surface. The edges are almost normal surfaces in the given isotopy class and connect two vertices found by the 'canonical' tightenings of the almost normal surface to each side to form normal surfaces. The graph corresponding to a given isotopy class is connected.

In the second part, we show that there is a simple criterion which guarantees that all the surfaces in the graph are incompressible. This is a necessary and sufficient condition.

Finally, the algorithm has been implemented by Nathan for ideal triangulations. The results are fascinating and we find information about the growth rates of isotopy classes of incompressible surfaces in terms of their genus in examples from the census of minimal ideal triangulations.

**Makoto Sakuma (Hiroshima University)**

*Kleinian groups generated by two parabolic transformations*

At a workshop held in Budapest in 2002, Ian Agol announced a classification of non-free Kleinian groups generated by two parabolic transformations, which generalizes the characterization, due to Colin Adams, of the hyperbolic 2-bridge links groups as cofinite volume Kleinian groups generated by two parabolic transformations.

I will talk about my joint project with Hirotaka Akiyoshi, Ken'ichi Ohshika, and John Parker to give a full proof to the announcement. I will also explain a conjectural picture of the space of Kleinian groups generated by two parabolic transformations, which was found through discussion with Gaven Martin, and a possible approach towards the proof of the conjecture.

**Stephan Tillmann (University of Sydney)**

*Structure of minimal triangulations*

The complexity of a 3-manifold is the minimum number of tetrahedra in a triangulation of the manifold. It was defined and first studied by Matveev in 1990. This complexity is generally difficult to compute, and various upper and lower bounds have been derived during the last two decades using fundamental group, homology or hyperbolic volume. The first sharp lower bounds leading to infinite families of closed manifolds for which the complexity was known are spherical space forms were given in joint work with Bus Jaco and Hyam Rubinstein in a series of three papers about ten years ago.

We recently revisited this work with Jonathan Spreer to completely characterise all closed 3-manifolds attaining our lower bounds on complexity of closed 3-manifolds and determined the first infinite family of examples with PSL geometry. Moreover, we extended our structure theory of minimal triangulations to those of cusped hyperbolic 3-manifolds, and as an application show that the monodromy ideal triangulations of the once-punctured torus bundles are minimal.

**Abigail Thompson (UC Davis)**

*Trisections and link surgeries*

A trisection of a closed, smooth orientable 4-dimensional manifold splits the manifold into three 4-dimensional handlebodies. The trisection induces framed links in the associated three dimensional boundaries. I'll discuss some natural conjectures that arise about integral surgeries on these links, and some modest progress on one of them. This is joint work with Rob Kirby.