(Im)possible emergent symmetry and conformal bootstrap

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earlier results are based on collaboration
with Tomoki Ohtsuki **Phys.Rev.Lett. 117 (2016)**

Symmetries in nature

- The great lesson from string theory

All the global continuous symmetries in nature are emergent

- Under which condition, discrete symmetry → continuous symmetry?

- How to realize continuous symmetry in lattice/condensed matter physics?

- If we find such “emergent symmetry”, what can we learn about the microscopic systems?

Two examples we discuss today

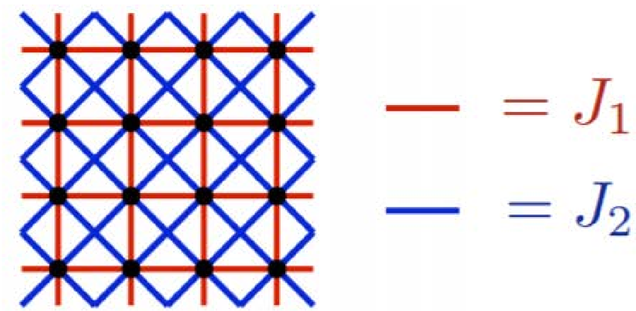
- Neel-VBS phase transition in frustrated spin system
  "king of emergent symmetry"
    - Emergent Poincare symmetry
    - Emergent gauge symmetry
    - Emergent conformal symmetry
    - Emergent continuous global symmetry from discrete lattice symmetry
    - Emergent SO(5) from SO(2)xSO(3) global symmetry(?)

- QCD chiral phase transition
    - Emergent(?) discrete symmetry at the chiral symmetry restoration
    - Further emergent continuum symmetry? Restoration of anomalous symmetry

Neel-VBS phase transition

Read-Sachdev, Senthil-Vishwanath-Balents-Sachdev-Fisher etc

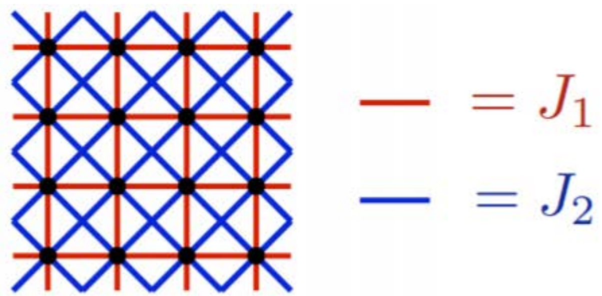
Neel-VBS phase transition



$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

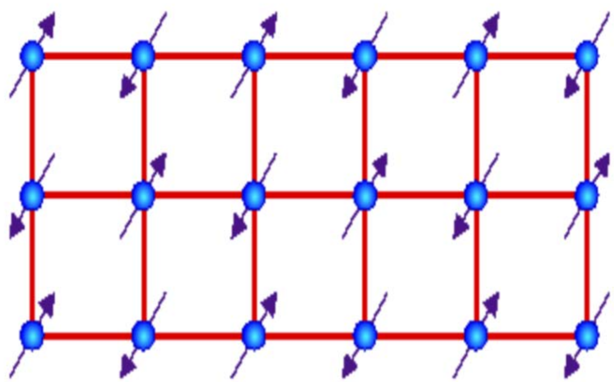
- Consider the frustrated quantum spin systems in 2+1 dim at zero temperature
- There exists a critical coupling such that
  - Emergent Poincare symmetry
  - Emergent gauge symmetry
  - Emergent conformal symmetry
  - Emergent continuum global symmetry from discrete lattice symmetry
  - Emergent SO(5) from SO(2)xSO(3) global symmetry(?)

Ground states in extreme limit 1



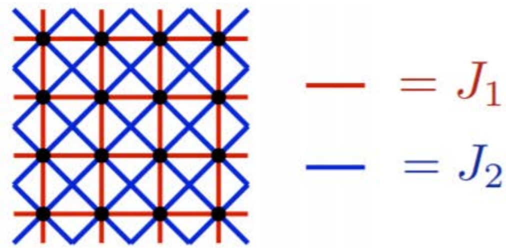
$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

- $J_1 \gg J_2$ anti-ferro or Neel state (SU(2) → U(1) symmetry breaking)



- Neel State is frustrated by J_2

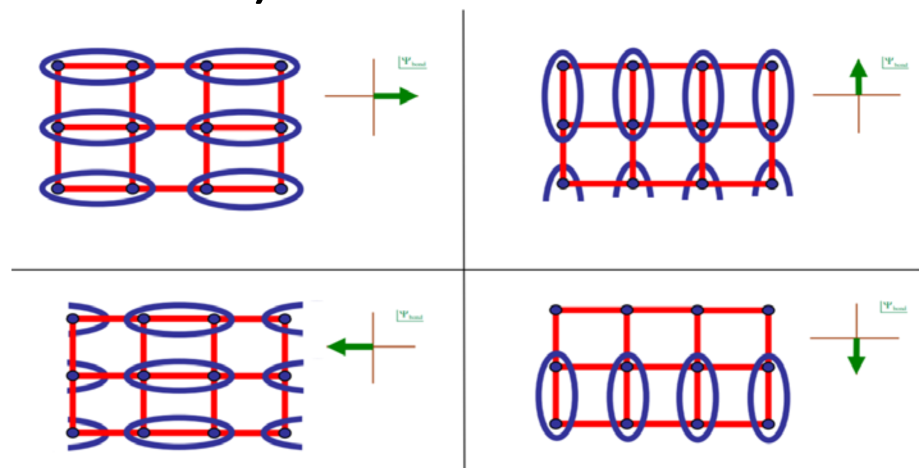
Ground states in extreme limit 2



$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

- $J_2 \gg J_1$ frustration cause Valence-Bond-State (VBS) ordering (Z4 is broken)

$$|\text{pair}\rangle = \frac{|+\rangle|-\rangle - |-\rangle|+\rangle}{\sqrt{2}}$$



- Lattice Z4 is enhanced to U(1) monopole symmetry in the continuum limit

Emergent Poincare symmetry

- Enough lattice symmetry to (heuristically) argue that near criticality dispersion is relativistic
- Finite scaling analysis of linear dispersion was numerically verified

- Rough idea: Poincare CFT is stable under Lorenz violating deformations except for spin 1 operator and anti-symmetric tensor operator
- Discrete symmetry like T and P combined with lattice symmetry forbids such deformations
- Different "speed of light" is renormalized to be the same because they are spin 2 and irrelevant.

EFT and emergent gauge symmetry

- Neel order parameter is decomposed into spin

$$\vec{S} = \bar{\Phi}^I \vec{\sigma}_{IJ} \Phi^J$$

- Gauge redundant U(1) → gauged linear sigma model
- "Emergent" U(1) gauge symmetry

$$S_{eff} = \int d^2x dt \left(F_{\mu\nu}^2 + (D_\mu \Phi_I)^2 + m^2 |\Phi_I|^2 + \lambda |\Phi_I|^4 \right)$$

- Deconfinement/confinement criticality
  - " $m^2 > 0$ " → confinement (= monopole condensation) → breaking of U(1) (or original Z4)
  - " $m^2 < 0$ " → deconfinement (=Higgs)→ breaking of SU(2)
- Unlike d=3+1 gauge theory, phases can be separated

VBS or monopole symmetry

- Monopole current: $$J^\mu = \epsilon^{\mu\nu\rho} F_{\nu\rho}$$
- Monopole operator with charge q
$$J^\mu = \partial^\mu a \ , \quad M_q \sim e^{iqa}$$
- Will identify monopole symmetry as VBS order
- To get the criticality under Z4, we have to ensure charge 4 monopole operators are irrelevant
- Possibility of Neel-VBS second order phase transition on the other lattice
  - Triangular lattice→ Z3 monopole operator must be irrelevant
  - Rectangular lattice → Z2 monopole operator must be irrelevant
- Large N or large q computation has been a hot issue

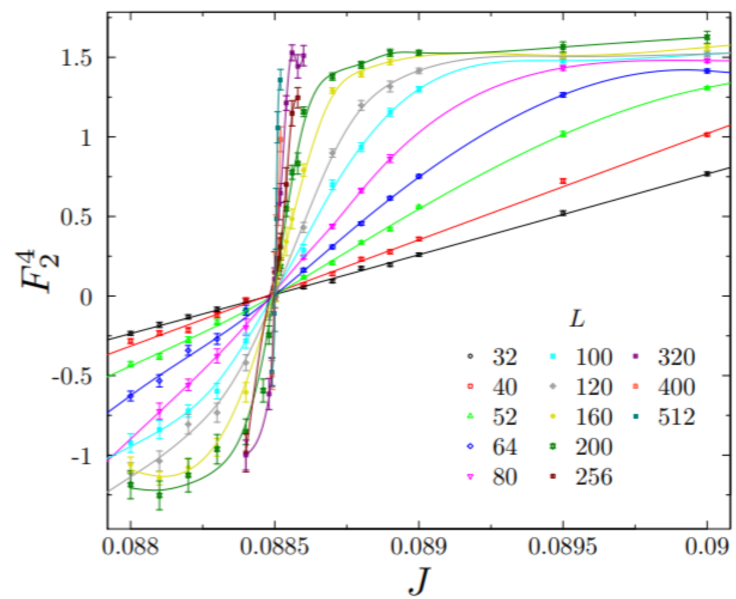
Further SO(5) enhancement?

- Nahum et al proposed a further symmetry enhancement of SO(3)xSO(2) → SO(5)
- Order parameter 2+3 = 5 $v_{i=1\dots5} = (\vec{S}, M_1, \bar{M}_1)$
- Numerical evidence: 2pt function scaling

$$\Delta_S \sim \Delta_{M_1} \sim 0.62$$

- Four-point function, Binder index

$$F_2^4 = \langle \vec{S}^4 - (M\bar{M})^2 \rangle$$



- Real thing or accident?
  - e.g. ABJM theory

Bootstrap study of emergent symmetry

- Setup: CFT with U(1) global symmetry. Let the lowest dimensional charge 1 operator O_1 with dimension Δ_1
- Bootstrap under which condition charge 2, 3, 4 operators may become irrelevant.

- Result: Necessary condition for the U(1) global symmetry enhancement (Nakayama-Ohtsuki)
  - From Z2 $\Delta_1 > 1.08$
  - From Z3 $\Delta_1 > 0.58$
  - From Z4 $\Delta_1 > 0.504$
- We have Neel-VBS in mind, but bound is universal

Bootstrap study of emergent symmetry

- Setup: CFT with U(1) global symmetry. Let the lowest dimensional charge 1 operator O_1 with dimension Δ_1
- Bootstrap equation to be studied:

$\langle O_i O_i^\dagger O_i O_i^\dagger \rangle$

$$\sum_{O\in O_i\times O_i^\dagger} |\lambda_{i\bar{i}O}|^2 F^{(-)ii,ii}_{\Delta_O,l_O} = 0,$$

$$\sum_{O\in O_i\times O_i} |\lambda_{iiO}|^2 F^{(\pm)ii,ii}_{\Delta_O,l_O} \pm \sum_{O\in O_i\times O_i^\dagger} |\lambda_{i\bar{i}O}|^2 (-)^l F^{(\pm)ii,ii} = 0,$$

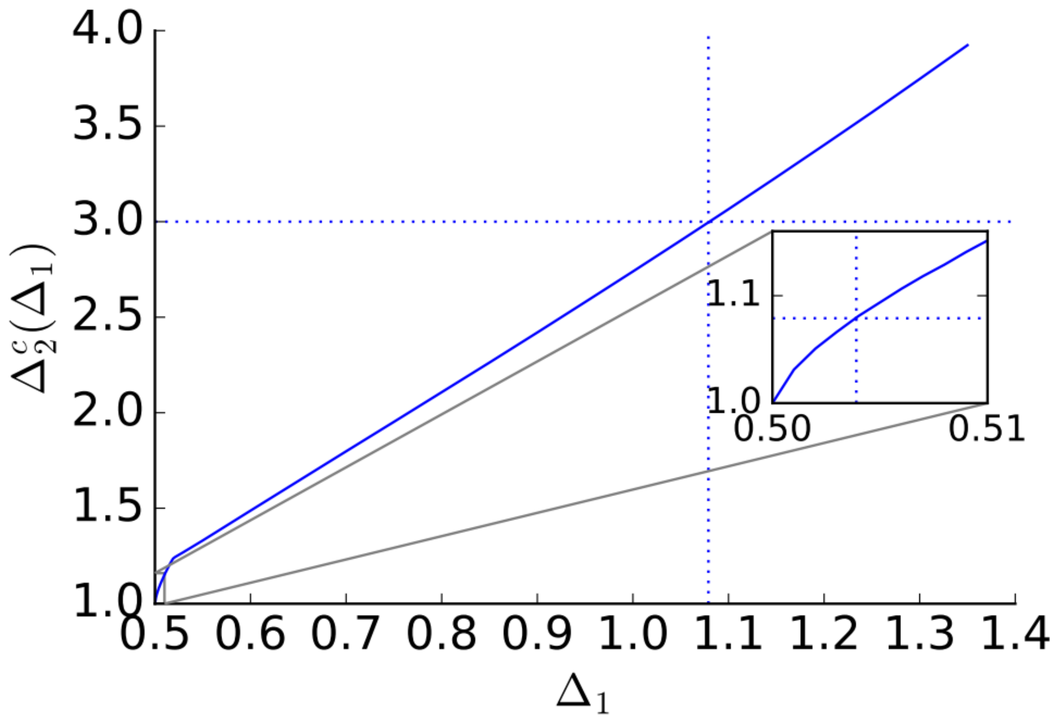
$\langle O_1 O_1^\dagger O_2 O_2^\dagger \rangle$

$$\sum_{O\in O_1\times O_2} |\lambda_{12O}|^2 F^{(\mp)12,21} \pm \sum_{O\in O_2\times O_1^\dagger} \lambda^*_{1\bar{1}O}\lambda_{2\bar{2}O}(-1)^{l_O} F^{(\mp)11,22} = 0$$

$$\sum_{O\in O_1\times O_2} |\lambda_{12O}|^2 (-1)^{l_O} F^{(\mp)21,21} \pm \sum_{O\in O_1\times O_2^\dagger} (-1)^{l_O} |\lambda_{12O}|^2 F^{(\mp)21,21} = 0$$

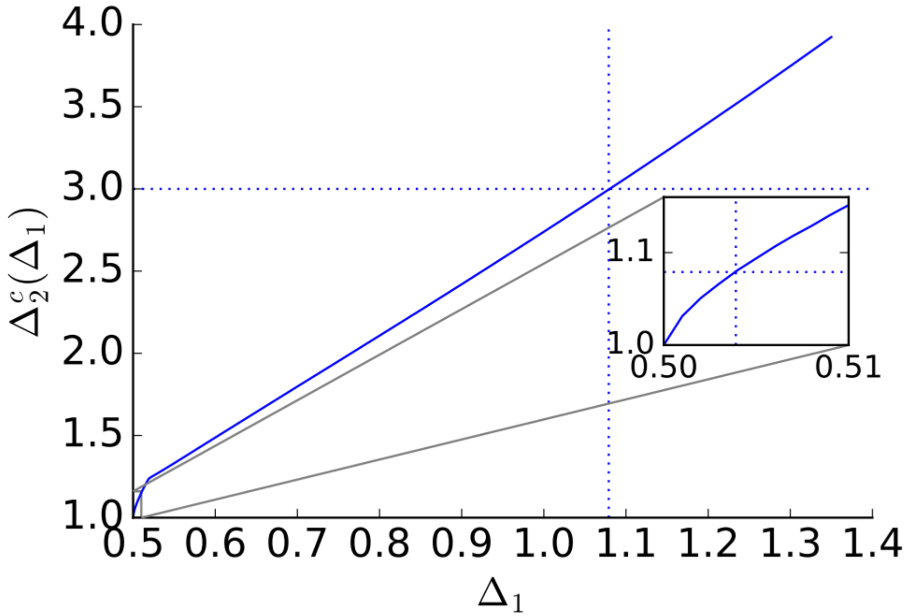
$$\sum_{O\in O_1\times O_2^\dagger} |\lambda_{1\bar{2}O}|^2 F^{(\mp)12,21} \pm \sum_{O\in O_2\times O_1^\dagger} \lambda^*_{1\bar{1}O}\lambda_{2\bar{2}O} F^{(\mp)11,22} = 0$$

Z2 enhancement



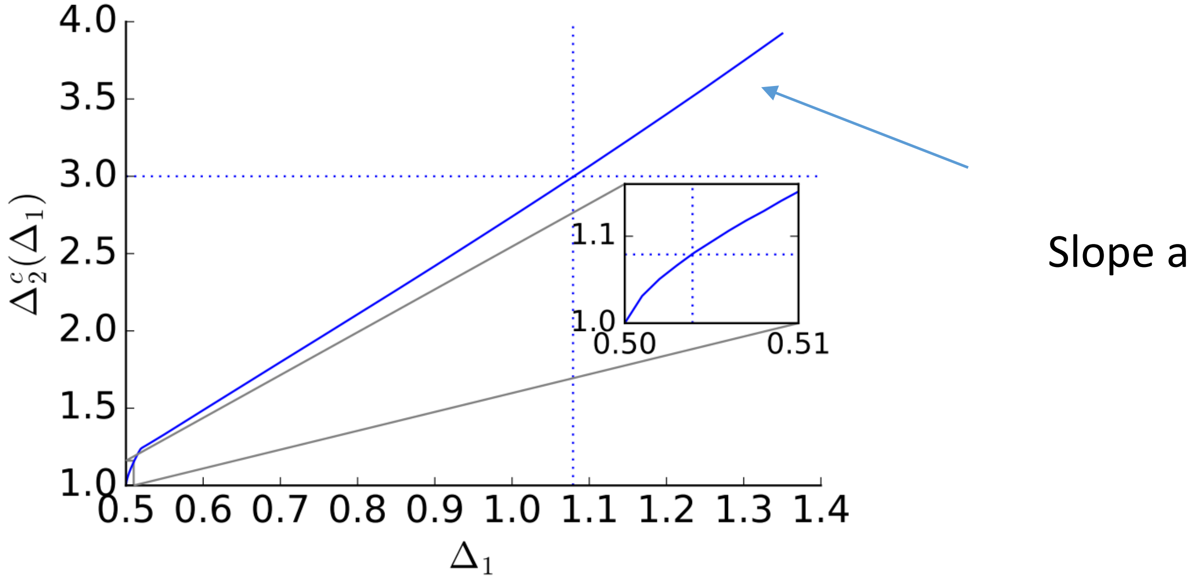
- Essentially the same bound in O(N) bootstrap by Poland et al, but in the larger Δ_1
- Necessary condition for the U(1) global symmetry enhancement $\Delta_1 > 1.08$
  - From Z2

Z4 enhancement



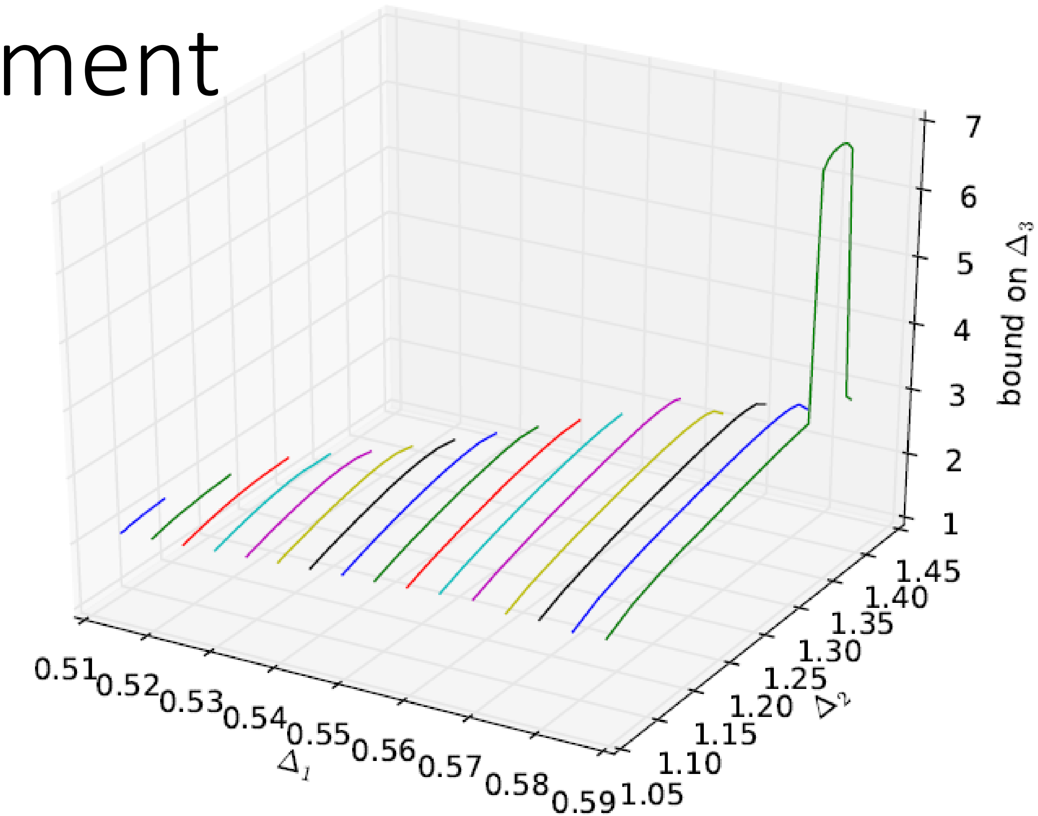
- Necessary condition for the U(1) global symmetry enhancement
  - From Z2 $\Delta_1 > 1.08$
  - From Z4 $\Delta_1 > 0.504$
- Simple mixed bootstrap does not improve the bound

Interlude or detour



- May expect the asymptotic bound is a straight line (Conjecture: can we prove it?)
- Repeat the analysis to get the bound $\Delta_{2^n} \sim q^{\log(a)/\log(2)}$
- In d=2 most likely a=4 (also numerically)
- In d=3 $\Delta_q \sim q^{1.6}$ (can we make it as strong as 1.5 as extremal RN-AdS black hole? e.g. Simeon's talk)
- d dependence?

Z3 enhancement

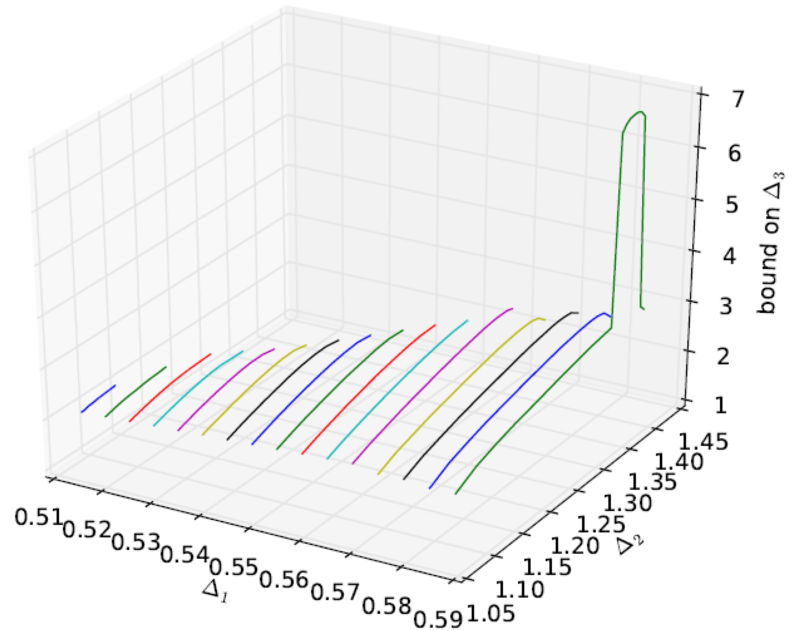


- Under the assumption
  - Only one relevant neutral operator
  - Charge four operator is irrelevant
- Necessary condition for the U(1) global symmetry enhancement
  - From Z3 $\Delta_1 > 0.58$

Comparison with lattice data 1

Necessary condition for the U(1) global symmetry enhancement

- From Z2 $\Delta_1 > 1.08$
- From Z3 $\Delta_1 > 0.58$
- From Z4 $\Delta_1 > 0.504$



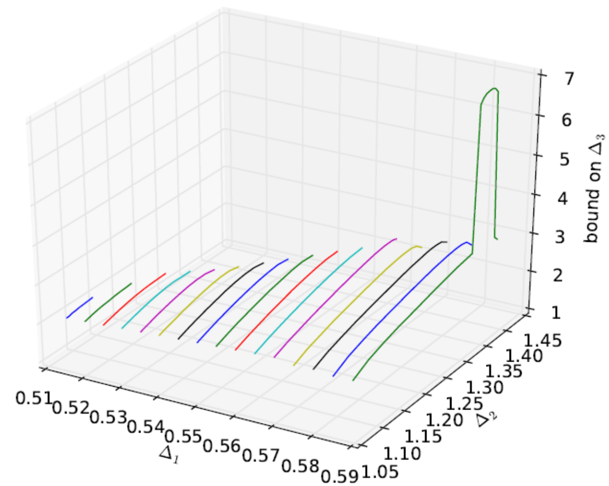
reference	Δ_0	Δ_1	Δ_2	Δ_3	Δ_4
JQ [42, 43]	no fixed point				
CDM [25, 44]	1.44(2)	0.579(8)	1.42(7)	2.80(3)	> 3
JQ [45, 46]	1.15(20)	0.64(4)		> 3	> 3
JQ [24]	1.31	0.68		> 3	> 3
JQ [23]		< 3	< 3	> 3	> 3
JQ [26]	1.53(5)	0.60(1)			> 3
large N [47]		0.63	1.50	2.55	3.77

Comparison with lattice data 2

- Study on more complex scalars (large N in mind)

Necessary condition for the U(1) global symmetry enhancement

- From Z2 $\Delta_1 > 1.08$
- From Z3 $\Delta_1 > 0.58$
- From Z4 $\Delta_1 > 0.504$



Lattice sim with SU(3) spin

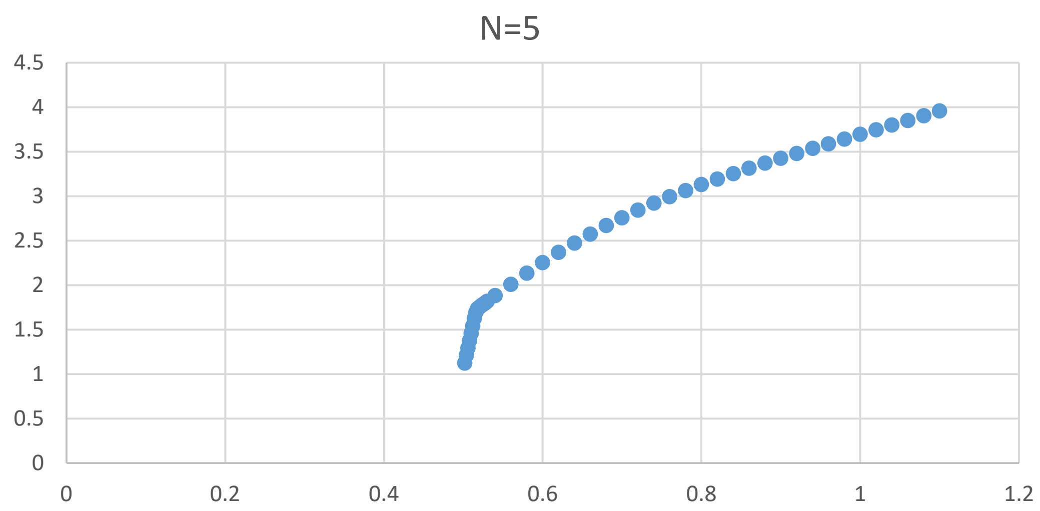
reference	Δ_0	Δ_1	Δ_2	Δ_3	Δ_4
JQ [24]	1.28	0.785	2.0	> 3	> 3
JQ [23]		< 3	< 3	> 3	> 3
JQ [26]	1.46(7)	0.71(2)			> 3
large N [47]		0.755	1.81	3.10	4.59

Lattice sim with SU(4) spin

reference	Δ_0	Δ_1	Δ_2	Δ_3	Δ_4
JQ [24]	1.60	0.865		> 3	> 3
JQ [23]		< 3	> 3	> 3	> 3
JQ [26]	1.57(4)	0.85(1)			> 3
large N [47]		0.880	2.12	3.64	5.40

SO(5) enhancement?

- Observed dimensions of putative “5” in Neel-VBS phase transition $\Delta_\Phi \sim \Delta_{M_1} \sim 0.62$
- Since the fixed point is reached under tuning only one parameter, there should exist only one SO(2)xSO(3) singlet operator → singlet operator in 5x5 must be irrelevant
- Bootstrap results



- Only compatible with (weak) first order phase transition

King of emergent symmetry

- Neel-VBS phase transition in frustrated spin system
  "king of emergent symmetry"

  - Emergent Poincare symmetry
  - Emergent gauge symmetry
  - Emergent conformal symmetry
  - Emergent continuous global symmetry from discrete lattice symmetry
  - Emergent SO(5) from SO(2)xSO(3) global symmetry(?)

- How to incorporate SU(2) and U(1) simultaneously?
- Identify CFT?

Finite temperature chiral phase transition in QCD

Is QCD chiral phase transition 1st or 2nd order?

- Consider idealistic QCD: SU(3) gauge theory with 2 massless Dirac fermions (in fundamental rep)

$$L = -\frac{1}{4g^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}^{La}D^{\mu}\sigma_{\mu}\psi_a^L + \bar{\psi}^{Ra}D^{\mu}\bar{\sigma}_{\mu}\psi_a^R$$

$$\psi_a^L \to (U\psi^L)_a \qquad \psi_a^R \to (V\psi^R)_a \qquad \langle\bar{\psi}\psi\rangle \sim \Lambda^3$$

$$SU(2)_L \times SU(2)_R \times U(1)_A \to SU(2)_V$$

- Will neglect $U(1)_V$ (never broken)
- $U(1)_A$ is anomalous $\partial^{\mu}J_{\mu}^{A} = g^2\mathrm{Tr}(F^{\mu\nu}\tilde{F}_{\mu\nu})$

- Finite temperature chiral phase transition

Landau theory

- Order parameter (neglecting anomaly)

$$SU(2)\times SU(2)\times U(1)\sim O(4)\times O(2)$$

$$\phi_a^\alpha\sim\bar{\psi}\psi$$

In Fund x Fund rep of O(4) x O(2)

- Effective free energy

$$\mathcal{H}=\partial_\mu\phi_a^\alpha\partial_\mu\phi_a^\alpha+(T-T_c)\phi_a^\alpha\phi_a^\alpha$$
$$+u(\phi_a^\alpha\phi_a^\alpha)^2+v(\phi_a^\alpha\phi_b^\alpha\phi_a^\beta\phi_b^\beta-\phi_a^\alpha\phi_a^\alpha\phi_b^\beta\phi_b^\beta)$$
$$+m_A(\phi_a^1\phi_a^1-\phi_a^2\phi_a^2)+c_A(\phi_a^1\phi_a^1\phi_a^1\phi_a^1+\cdots)$$

O(4)x O(2) symmetric

Anomaly

- Non-zero m_A → May integrate out half ϕ_a^1
- → O(4) vector model in d=3 → 2nd order transition (with known exponents e.g. from conformal bootstrap)

Effective/accidental $U(1)_A$ restoration?

Some argue $U(1)_A$ is restored (or emerges) above Tc

- At vacuum $U(1)_A$ is broken down to Z2 via instanton (non-perturbative) effect
- Above Tc, such effects may be negligible (Pisarski-Wilczek, Cohen)

- Aoki et al argued that at least Z4 (or extra Z2) out of

$U(1)_A$ is shown to be restored in the meson correlation function

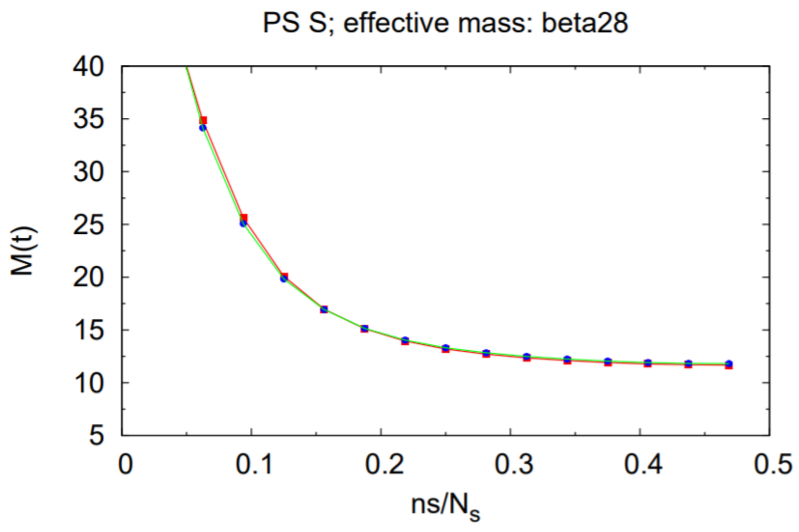
Z4 (extra Z2) recovery in lattice simulation

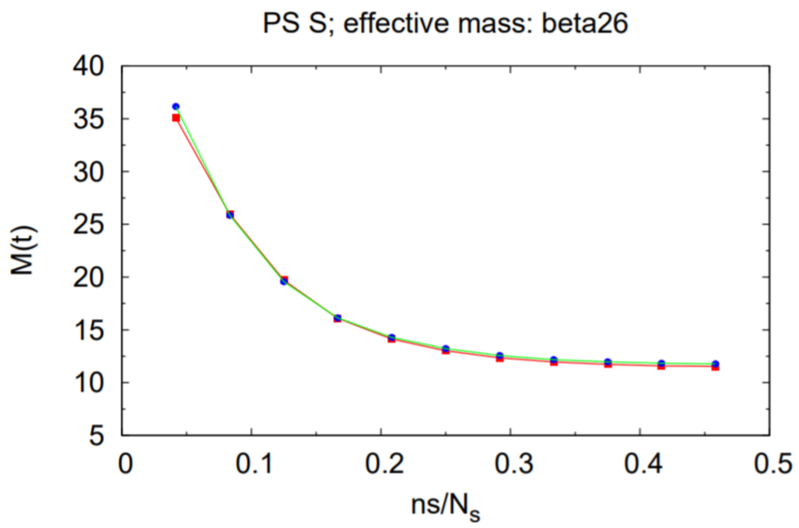
$$\sum_x \langle \bar{\psi}\psi(t,x)\bar{\psi}\psi(0,0)\rangle$$

vs

$$\sum_x \langle \bar{\psi}\gamma_5\psi(t,x)\bar{\psi}\gamma_5\psi(0,0)\rangle$$

Wilson fermion

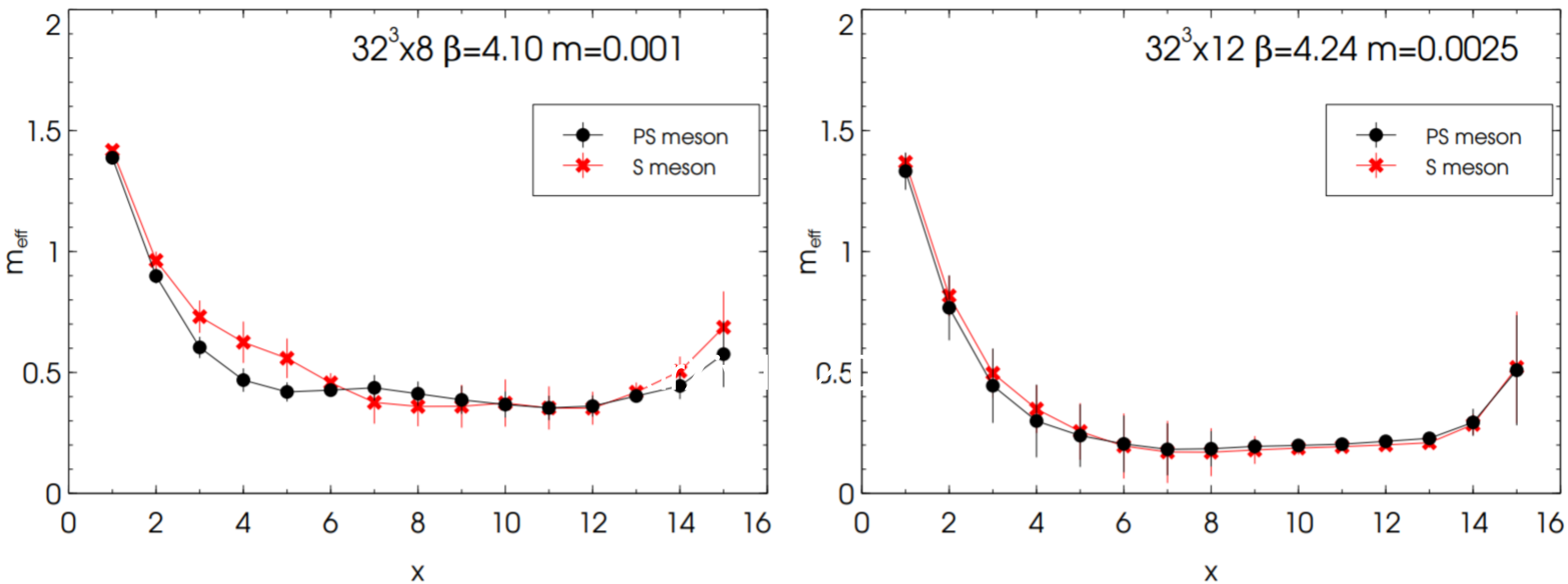




Ishikawa-Iwasaki-Nakayama-Yoshie

arxiv:1706.08872

Z4 (extra Z2) recovery in lattice simulation



Mobius domain wall fermion

Tomiya et al **arXiv:1612.01908**

"Aoki-Fukaya-Taniguchi argument"

- Introduce "supurious mass" m^2

1 : free energy is analytic wrt m above Tc

2: Dirac eigenvalue distribution $D^\mu \gamma_\mu \psi_i = \lambda \psi_i$ is analytic at $\lambda = 0$

- Macroscopic evaluation of free energy

$$f = f_0 + c_0(|m_u|^2 + |m_d|^2) + c_a(m_u m_d + \bar{m}_u \bar{m}_d) + \cdots$$

- Microscopic evalulation shows $c_a = 0$

$$\lim_{m_u \to 0} \frac{\partial f}{\partial m_u} = c_a m_d + \cdots$$

$$= \lim_{m_u \to 0} \int d^4x \langle \bar{u}(x) u(x) \rangle = \lim_{m_u \to 0} \int d\lambda \frac{m_u \langle \rho(\lambda) \rangle}{m_u^2 + \lambda^2}$$

$$= \langle \rho(0) \rangle = m_d^2 + \cdots$$

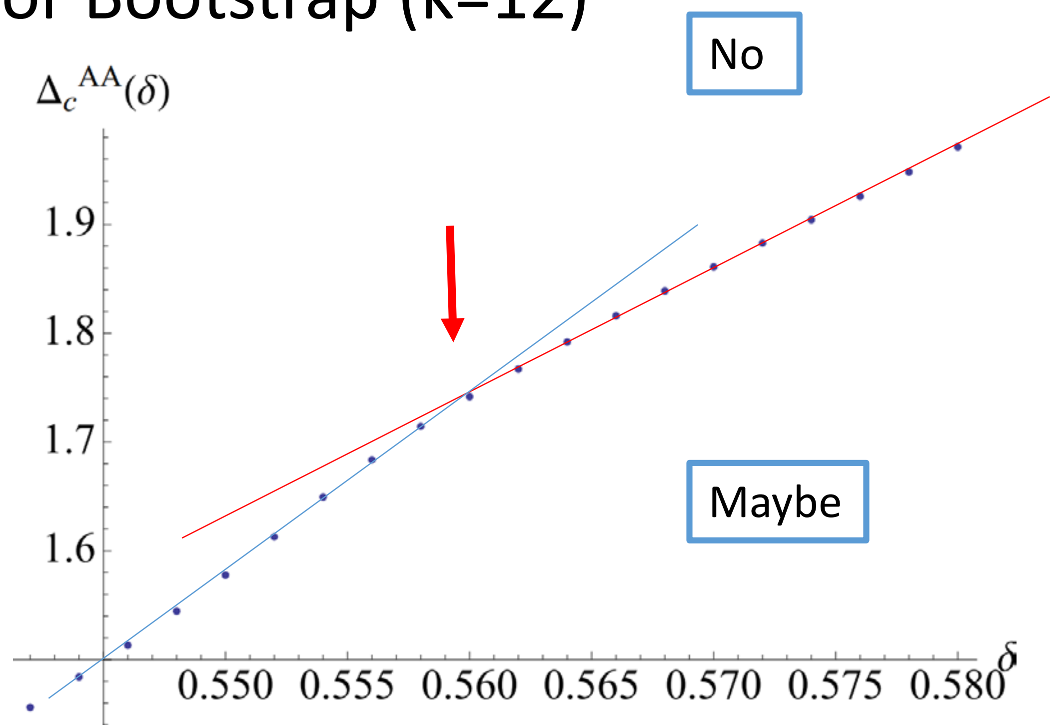
- One expects at least extra Z2 out of $U(1)_A$ is restored above Tc

Bootstrap study of emergent symmetry

- Setup: Bootstrap CFT with O(4) x O(2) symmetry in d=3
- Can we identify the would be fixed point (with $U(1)_A$ restoration)?
- Under which condition, it is stable?
    - Is Z4 (or extra Z2) enough?

O(4) x O(2) collinear fixed point

- AA sector Bootstrap (k=12)



	Δ_ϕ	Δ_{SS}	Δ_{ST}	Δ_{TS}	Δ_{TT}	Δ_{AA}
bootstrap	0.558(4)	1.52(5)	0.82(2)	1.045(3)	1.26(1)	1.71(6)
$\overline{\mathrm{MS}}$	0.56(3)	1.68(17)	1.0(3)	1.10(15)	1.35(10)	1.9(1)
MZM	0.56(1)	1.59(14)	0.95(15)	1.25(10)	1.34(5)	1.90(15)

Collinear fixed points → 2[nd] order phase transition in QCD

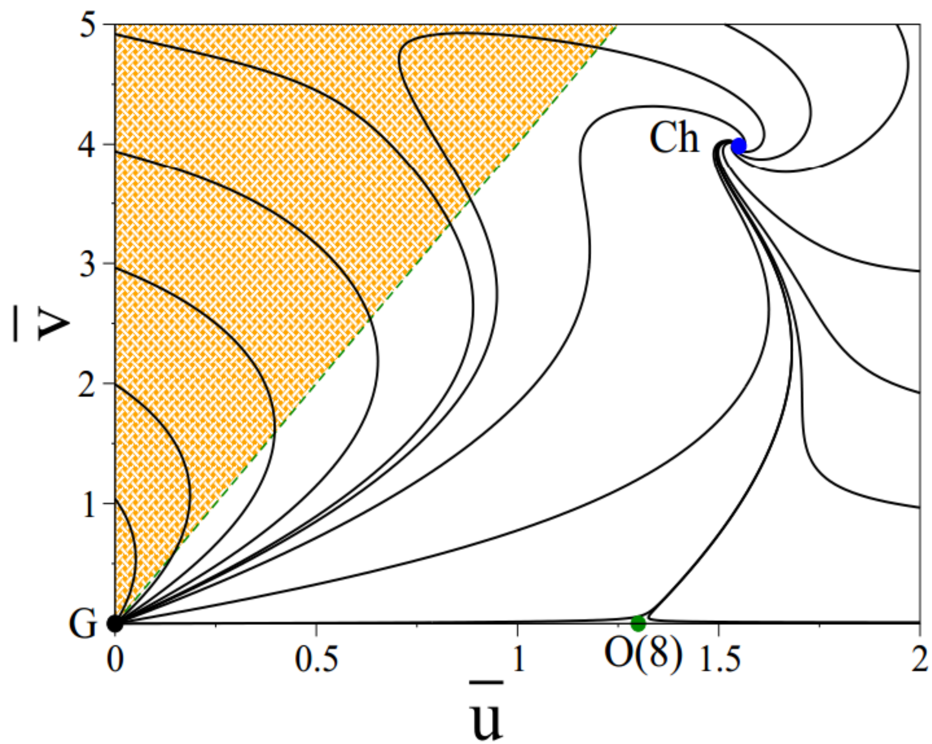
5-loop beta function (Calabrese, Vicari, e.g. 1309.5446)

$$\mathcal{H} = \partial_\mu \phi_a^\alpha \partial_\mu \phi_a^\alpha + u(\phi_a^\alpha \phi_a^\alpha)^2 + v(\phi_a^\alpha \phi_b^\alpha \phi_a^\beta \phi_b^\beta - \phi_a^\alpha \phi_a^\alpha \phi_b^\beta \phi_b^\beta)$$

$$\begin{aligned}
\beta_u(u,v) = & -u + 4u^2 + 4uv + \tfrac{3}{2}v^2 - \tfrac{57}{8}u^3 - 11u^2v - \tfrac{61}{8}uv^2 - 3v^3 + \tfrac{93}{8}u^4\zeta(3) + \tfrac{389}{16}u^4 + 24u^3v\zeta(3) + \tfrac{975}{16}u^3v \\
& + \tfrac{99}{4}u^2v^2\zeta(3) + \tfrac{9347}{128}u^2v^2 + 18uv^3\zeta(3) + 45uv^3 + 6v^4\zeta(3) + \tfrac{1197}{128}v^4 - \tfrac{1885}{16}u^5\zeta(5) - \tfrac{3119}{32}u^5\zeta(3) + \tfrac{31}{120}\pi^4u^5 \\
& - \tfrac{51759}{512}u^5 - 340u^4v\zeta(5) - \tfrac{1183}{4}u^4v\zeta(3) + \tfrac{161}{240}\pi^4u^4v - \tfrac{10449u^4}{32}v - \tfrac{3905}{8}u^3v^2\zeta(5) - \tfrac{6849}{16}u^3v^2\zeta(3) + \tfrac{353}{480}\pi^4u^3v^2 \\
& - \tfrac{391151}{768}u^3v^2 - \tfrac{895}{2}u^2v^3\zeta(5) - 377u^2v^3\zeta(3) + \tfrac{8}{15}\pi^4u^2v^3 - \tfrac{42919}{96}u^2v^3 - \tfrac{1875}{8}uv^4\zeta(5) - \tfrac{3049}{16}uv^4\zeta(3) \\
& + \tfrac{31}{96}\pi^4uv^4 - \tfrac{12697}{64}uv^4 - 50v^5\zeta(5) - \tfrac{325}{8}v^5\zeta(3) + \tfrac{5}{48}\pi^4v^5 - \tfrac{1097}{32}v^5 + \tfrac{646947}{512}u^6\zeta(7) + \tfrac{333739}{256}u^6\zeta(5) \\
& - \tfrac{3}{64}u^6\zeta(3)^2 + \tfrac{333239}{512}u^6\zeta(3) - \tfrac{1885}{4032}\pi^6u^6 - \tfrac{6827}{2560}\pi^4u^6 + \tfrac{121665}{256}u^6 + \tfrac{146853}{32}u^5v\zeta(7) + \tfrac{158151}{32}u^5v\zeta(5) \\
& - \tfrac{507}{32}u^5v\zeta(3)^2 + \tfrac{320791}{128}u^5v\zeta(3) - \tfrac{6625}{4032}\pi^6u^5v - \tfrac{37481}{3840}\pi^4u^5v + \tfrac{494921}{256}u^5v + \tfrac{4266675}{512}u^4v^2\zeta(7) + \tfrac{1144635}{128}u^4v^2\zeta(5) \\
& - \tfrac{7017}{256}u^4v^2\zeta(3)^2 + \tfrac{4795927}{1024}u^4v^2\zeta(3) - \tfrac{9615}{3584}\pi^6u^4v^2 - \tfrac{504343}{30720}\pi^4u^4v^2 + \tfrac{7852881}{2048}u^4v^2 + 9702u^3v^3\zeta(7) + \tfrac{79415}{8}u^3v^3\zeta(5) \\
& - \tfrac{3}{4}u^3v^3\zeta(3)^2 + \tfrac{171801}{32}u^3v^3\zeta(3) - \tfrac{155}{56}\pi^6u^3v^3 - \tfrac{8173}{480}\pi^4u^3v^3 + \tfrac{6814435}{1536}u^3v^3 + \tfrac{3658095}{512}u^2v^4\zeta(7) + \tfrac{1773961}{256}u^2v^4\zeta(5) \\
& + \tfrac{2777}{128}u^2v^4\zeta(3)^2 + \tfrac{1942077}{512}u^2v^4\zeta(3) - \tfrac{94645}{48384}\pi^6u^2v^4 - \tfrac{182363}{15360}\pi^4u^2v^4 + \tfrac{36147287}{12288}u^2v^4 + \tfrac{189189}{64}uv^5\zeta(7) \\
& + \tfrac{44351}{16}uv^5\zeta(5) + \tfrac{103}{8}uv^5\zeta(3)^2 + \tfrac{189841}{128}uv^5\zeta(3) - \tfrac{2585}{3024}\pi^6uv^5 - \tfrac{1647}{320}\pi^4uv^5 + \tfrac{2121643}{2048}uv^5 \\
& + \tfrac{265041}{512}v^6\zeta(7) + \tfrac{61459}{128}v^6\zeta(5) + \tfrac{81}{64}v^6\zeta(3)^2 + \tfrac{246291}{1024}v^6\zeta(3) - \tfrac{335}{2016}\pi^6v^6 - \tfrac{3125}{3072}\pi^4v^6 + \tfrac{2538035}{16384}v^6, \qquad (1
\end{aligned}$$

$$\begin{aligned}
\beta_v(u,v) = & -v + 3uv + 2v^2 - \tfrac{61}{8}u^2v - 11uv^2 - \tfrac{27}{8}v^3 + \tfrac{1349}{64}u^3v + \tfrac{1451}{32}u^2v^2 + \tfrac{575}{16}uv^3 + \tfrac{347}{32}v^4 + \tfrac{33}{2}u^3v\zeta(3) \\
& + 36u^2v^2\zeta(3) + 24uv^3\zeta(3) + \tfrac{9}{2}v^4\zeta(3) - \tfrac{49815}{512}u^4v + \tfrac{29}{64}\pi^4u^4v - \tfrac{27835}{96}u^3v^2 + \tfrac{163}{120}\pi^4u^3v^2 - \tfrac{272945}{768}u^2v^3 + \tfrac{22}{15}\pi^4u^2v^3 \\
& - \tfrac{6635}{32}uv^4 + \tfrac{53}{80}\pi^4uv^4 - \tfrac{365}{8}v^5 + \tfrac{1}{10}\pi^4v^5 - \tfrac{3765}{32}u^4v\zeta(3) - \tfrac{691}{2}u^3v^2\zeta(3) - \tfrac{6111}{16}u^2v^3\zeta(3) - \tfrac{1507}{8}uv^4\zeta(3) - \tfrac{567}{16}v^5\zeta(3 \\
& - \tfrac{2625}{16}u^4v\zeta(5) - 480u^3v^2\zeta(5) - \tfrac{2115}{4}u^2v^3\zeta(5) - \tfrac{1045}{4}uv^4\zeta(5) - \tfrac{795}{16}v^5\zeta(5) + \tfrac{445355}{1024}u^5v - \tfrac{58367}{15360}\pi^4u^5v \\
& - \tfrac{12115}{16128}\pi^6u^5v + \tfrac{209163}{128}u^4v^2 - \tfrac{109087}{7680}\pi^4u^4v^2 - \tfrac{7535}{2688}\pi^6u^4v^2 + \tfrac{16837765}{6144}u^3v^3 - \tfrac{81491}{3840}\pi^4u^3v^3 - \tfrac{8455}{2016}\pi^6u^3v^3 \\
& + \tfrac{3808447}{1536}u^2v^4 - \tfrac{30289}{1920}\pi^4u^2v^4 - \tfrac{38005}{12096}\pi^6u^2v^4 + \tfrac{9331663}{8192}uv^5 - \tfrac{1485}{256}\pi^4uv^5 - \tfrac{28555}{24192}\pi^6uv^5 + \tfrac{825245}{4096}v^6 - \tfrac{1285}{1536}\pi^4v^6 \\
& - \tfrac{5}{28}\pi^6v^6 + \tfrac{395479}{512}u^5v\zeta(3) + \tfrac{734983}{256}u^4v^2\zeta(3) + \tfrac{282653}{64}u^3v^3\zeta(3) + \tfrac{56043}{16}u^2v^4\zeta(3) + \tfrac{736561}{512}uv^5\zeta(3) + \tfrac{63485}{256}v^6\zeta(3) \\
& + \tfrac{2499}{128}u^5v\zeta(3)^2 + \tfrac{4305}{64}u^4v^2\zeta(3)^2 + \tfrac{405}{4}u^3v^3\zeta(3)^2 + \tfrac{2771}{32}u^2v^4\zeta(3)^2 + \tfrac{2579}{64}uv^5\zeta(3)^2 + \tfrac{231}{32}v^6\zeta(3)^2 + \tfrac{105231}{64}u^5v\zeta(5) \\
& + \tfrac{390003}{64}u^4v^2\zeta(5) + \tfrac{295491}{32}u^3v^3\zeta(5) + \tfrac{457495}{64}u^2v^4\zeta(5) + \tfrac{5647}{2}uv^5\zeta(5) + \tfrac{29035}{64}v^6\zeta(5) + \tfrac{472311}{256}u^5v\zeta(7) \\
& + \tfrac{218295}{32}u^4v^2\zeta(7) + \tfrac{1325205}{128}u^3v^3\zeta(7) + \tfrac{1030617}{128}u^2v^4\zeta(7) + \tfrac{816291}{256}uv^5\zeta(7) + \tfrac{16317}{32}v^6\zeta(7). \qquad (1
\end{aligned}$$

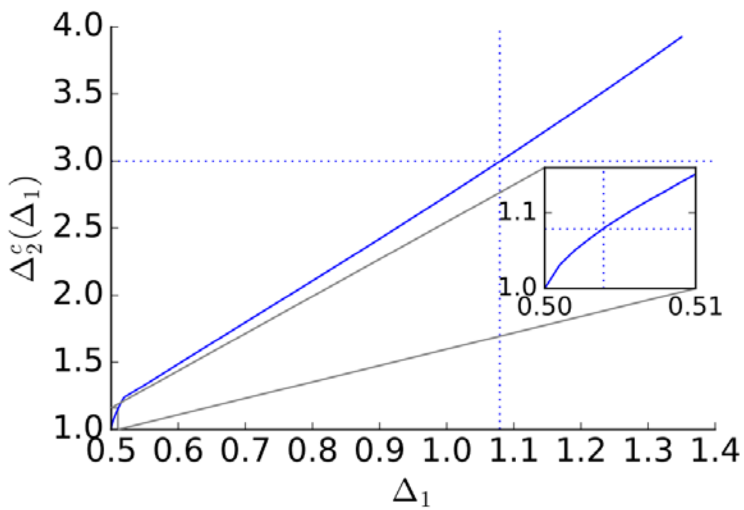
Resumed RG flow?



- The spiral behavior looks peculiar
  - Complex RG eigenvalue → non-unitary? Removable by wavefunction renormalization? (e.g. scale vs conformal)

Z2 stability?

- Does this fixed point stable under Z4 (extra Z2)?
- The question we asked before:



- Exists O(4) singlet charge 2 operator with $\Delta = 0.82$
- → Exists O(4) singlet charge 4 relevant operator

- Unfortunately the fixed point is unstable

Surviving scenarios

(A) $U(1)_A$ is (somehow) completely recovered
→ O(4) x O(2) 2nd order phase transition

$$\Delta_\epsilon = 1.52$$

(B) Z4 (or Z2) is recovered
→ Cannot reach O(4) x O(2) fixed point
→ Probably first order phase transition

(C) $U(1)_A$ is broken
→ O(4) 2nd order phase transition $\Delta_\epsilon = 1.665$