

Super perfect numbers and Mersenne perfect numbers(abstract)

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February 27, 2019

If a positive integer a satisfies $\sigma(a) - 2a = -m$, then we say that a is a perfect number with translation parameter m .

If α is a perfect number, then it is written as $2^e q$.

We shall study 2^e and q , independently.

In general, If $q = 2^{e+1} - 1 + m$ is prime then $a = 2^e$ satisfies the following equalities. positive integers a and A satisfy $A = \sigma(a) + m$, $\sigma(A) = 2a + m$. a is called a super perfect number with translation m .

A is said to be a partner of a .

Super perfect number was introduced by D.Suryanaryana when $m = 0$, in 1969. He proves if a super perfect number a is even, then a turns out to be 2^e such that $p = 2^{e+1} - 1$ is prime (Mersenne prime).

In general if odd primes p, q satisfy $q = ap + b$, where integers $a > 0, b$ have the properties (1) $p + q \equiv 1 \pmod{4}$, (2) a, b are relatively primes, then p, q are said to be super twin primes by Hiroto Takahashi. In 2019, the prime part q of the perfect number $\alpha = 2^e q$ is studied.

If $a = 2^{e+1} - 1 + m$ is prime, then $\sigma(a) = a + 1$. Thus $\sigma(a) = a + 1 = 2^{e+1} + m$.

Letting A be $\sigma(a) - m$, we get $A = 2^{e+1}$.

From $a + 1 = 2^{e+1} + m$, it follows that

$$\sigma(A) = 2^{e+2} - 1 = 2 * 2^{e+1} - 1 = 2a - 2m + 1.$$

Definition 1 A positive integer a satisfying the above equation is said to be super Mersenne perfect number with translation m , and A a partner of a .